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Optimum Allocation of Winter Wheat Samples in the Western Two-Thirds of Kansas

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During the 1964 Wheat Objective Yield Survey, enumerators recorded the amount of time and mileage required to make their counts and measurements. From this data and the survey information, an optimum allocation of wheat samples in the Western two-thirds of Kansas was made.

The objectives were (1) to compare the optimum number of samples per segment and the optimum number of segments by strata (Crop Reporting Districts) with the 1964 Objective Yield Survey and (2) to obtain an estimate of the time and mileage costs for wheat objective yield work between and within segments.

Methods and Procedures

Kansas was chosen for this study because the existing data were the most suitable for the method of analysis. It had more segments with wheat and more wheat samples per segment than other states. The western 6 Crop Reporting Districts were used since this is where most of the State's wheat is grown.

The information recorded on the B-3 Forms was used in the analysis as they contained the pre-harvest data. **Variance** were computed based on the variable, gross yield, which reflects most of the other variables that could have been used, such as net yield, plant counts, head counts, and head weights. Another reason for using gross yields was they were already computed for each sample which made the data readily available.

An analysis of variance (AOV) using unequal subclass numbers was computed for each of six districts. From the AOV's estimates of the within and between variances were obtained. (See Appendix I). The cost data were summarized from the enumerators' time and mileage logs.

The optimum number of samples per segment for the h^{th} district was computed using the following formula:

$$\text{opt } \bar{n}_h = \sqrt{\frac{S^2_{wh} \cdot C_{bh}}{S^2_{bh} \quad C_{wh}}}$$

Where S^2_{wh} is the estimated within segment variance for the h^{th} district and S^2_{bh} is the estimated between segment variance for the h^{th} district. C_{bh} is the average between segment cost and C_{wh} is the average within segment cost for the h^{th} district.

To obtain the optimum number of segments per district based on a fixed cost of about $1825 the following formula was used:

$$\text{opt } m_h \text{ 1/} = \frac{C \cdot N_h \cdot \frac{S_{wh}}{\bar{n}_h} \cdot \sqrt{C_{wh}}}{\sum_h^L (C_{bh} + C_{wh} \cdot \bar{n}_h) \cdot N_h \cdot \frac{S_{wh}}{\bar{n}_h} \cdot \sqrt{C_{wh}}}$$

Where $C = \sum_h^L C_{bh} M_h + \sum_h^L C_{wh} M_h \bar{n}_h$ = $1825 for this study. N_h is the estimated total number of acres of wheat in the h^{th} district. S_{wh} is the square root of the within segment variance for the h^{th} district and $\bar{n}_h$ is the optimum average number of samples per segment. C_{bh} and C_{wh} are the same as previously defined and h = 1, 2, 3, ... L.

Results:

On the average it took enumerators 77.2 minutes to complete a Form B-3. (See Table 1). The enumerators drove an average of 7.8 miles in 23 minutes inside each segment. Enumerators spent an average of 92.2 minutes driving 54.7 miles between segments. Only one or two days of per diem were claimed for the survey period. This was not included in the cost estimates.

1/ Hansen, Morris, A., Hurwitz, William N. and Madow, William G.; Sample Survey Methods and Theory. Vol. I, page 3-26, John Wiley and Sons, Inc. New York, 1962.

The 1964 Objective Yield Survey was allocated 215 samples in the Western two-thirds of Kansas. In the analysis only the usable samples were included. The average number per segment or $\bar{n}_h$ for the 1964 survey was derived by taking the number of usable samples and dividing by the number of segments in the district. To get the total number of samples per district, $\bar{n}_h$ was multiplied by the number of segments involved. The derived number of samples in the 1964 survey for the six districts was 208.8 (See Table 2). This was 22.47 more samples than the optimum allocation required.

The variance of the estimate $\bar{Y}$ for the 1964 survey was 0.426 as compared with 0.394 for the optimum allocation. The optimum allocation shows an 8 percent reduction in variance with about 22 fewer samples for the same cost.

The optimum allocation also indicated District VII should have 45 segments. Since the June Enumerative Survey, or the area frame allocated, only contained 42 segments, the other 3 segments were allocated to the other 5 districts for this anlaysis.

Conclusions:

The optimum allocation indicated fewer samples were needed especially in Districts VII and VIII. In total, more segments should be included in the survey but fewer sample fields per segment are required. Also, more segments in the area frame should be allocated to District VII to provide a better frame for the Wheat Objective Yield Survey.

Basically, the optimum allocation study showed an 8 percent reduction in variance for the same cost using fewer samples.

The optimum number of samples per segment ranged from 1 to 3 samples. Generally, 2 samples per segment would be sufficient for all districts in an operational survey in the Western two-thirds of Kansas.

For this study and for most studies of this type the cost relationships are assumed linear. Linear cost relationships may not always exist. This suggests as more cost data are collected, an extensive cost analysis study needs to be conducted.

Table 1.--Winter Wheat - the within and between segment time and mileage for the 1964 B-3 visit of the Wheat Objective Yield Survey by district for the western two-thirds of Kansas

District	Number of segments	Number of samples	Within segment time and mileage						Between segment time and mileage			
			Total time Form B-3	Average per Form B-3 1/	Travel time	Average travel time per segment 1/	Miles driven in segment 1/	Average miles in segment 1/	Travel time	Average travel time per segment 1/	Miles driven between segments	Average miles between segments 1/
			(Minutes)	(Minutes)	(Min.)	(Minutes)	(Miles)	(Miles)	(Min.)	(Minutes)	(Miles)	(Miles)
I	15	29	1670	57.6	247	16.5	68	4.5	2305	153.7	1239	82.6
II	17	30	2001	77.0	506	33.7	202	13.5	1614	107.6	1054	70.3
IV	16	23	1607	73.0	336	22.4	94	6.3	1758	117.2	1022	68.1
V	25	39	2471	70.6	316	13.2	75	3.1	1581	65.9	937	39.0
VII	23	47	3203	76.3	817	35.5	323	14.0	1512	65.7	985	42.8
VIII	27	47	4254	98.9	444	18.5	148	6.2	1927	80.3	1109	46.2
Total	123	215	15206	77.2	2666	23.0	910	7.8	10697	92.2	6346	54.7

1/ Based on usable samples in survey.

Table 2.--Winter Wheat - allocation of the 1964 Objective Yield Survey for the western two-thirds of Kansas

District	Wheat for grain N_h	Wheat per segment $\bar{N}_h$	Samples per segment $\bar{n}_h$ 1/	Segments M_h	Segments in sample m_h	Total samples $\bar{n}_h \times m_h$	Between segment variance	Within segment variance	Variance $\bar{Y}_h$	Cost
	(Acres)	(Acres)	(Number)	(Number)	(Number)	(Number)				(Dollars)
I	1,152,292	278.3	1.93	38	15	28.95	90.87	91.69	4.917	272.48
II	1,216,621	101.2	1.73	49	17	29.41	50.45	64.41	2.698	299.29
IV	907,627	374.9	1.47	37	16	23.52	61.67	90.32	3.848	263.47
V	1,572,241	158.5	1.46	53	25	36.50	43.49	59.87	1.693	269.06
VII	1,881,668	390.0	1.86	42	23	42.09	98.67	48.77	2.283	320.76
VIII	1,876,869	202.7	1.79	55	27	48.33	74.40	39.74	1.806	400.49
Total	8,607.318	250.9	1.70	274	123	208.80			0.426	1825.55

1/ Based on usable samples.

Table 3.--Winter Wheat - optimum allocation of objective yield samples for the western two-thirds of Kansas

District	Wheat for grain N_h	Wheat per segment N_h	Samples per segment $\bar{n}_h$	Segments M_h	Segments in sample m_h	Total samples $n_h \times m_h$	Between segment variance	Within segment variance	Variance $\bar{Y}_h$	Cost
	(Acres)	(Acres)	(Number)	(Number)	(Number)	(Number)				(Dollars)
I	1,152,292	278.3	2.22	38	11.97	26.57	90.87	91.69	6.287	226.59
II	1,216,621	101.2	1.76	49	16.97	29.87	50.45	64.41	2.690	300.93
IV	907,627	374.9	2.12	37	11.48	24.34	61.67	90.32	4.855	215.98
V	1,572,241	158.5	1.57	53	20.94	32.88	43.49	59.87	1.976	233.01
VII	1,881,668	390.0	.86	42	42.00	36.12	98.67	48.77	1.350	414.62
VIII	1,876,869	202.7	.94	55	38.88	36.55	74.40	39.74	1.330	433.94
Total	8,607,318	250.9	1.31	274	142.24	186.33			0.394	1825.07

Appendix I

Winter Wheat - AOV and opt. $\bar{n}_h$ for Objective Yield Samples in Kansas Crop Reporting Districts I and II

District I
AOV

Source	D.F.	S.S.	M.S.	E.M.S.
Between segments	14	3708.63	264.90	$S^2_{wI} + 1.906\ S^2_{bI}$
Within segments	14	1283.69	91.69	S^2_{wI}
Total	28	4992.32		

$$S^2_{bI} = \frac{264.90 - 91.69}{1.906} = 90.87$$

$$C_{bI} = \$13.07 \quad C_{wI} = \$2.64$$

$$\text{opt. } \bar{n}_I = \sqrt{\frac{91.69}{90.87} \times \frac{13.07}{2.64}} = 2.22$$

District II
AOV

Source	D.F.	S.S.	M.S.	E.M.S.
Between segments	14	2108.84	150.63	$S^2_{wII} + 1.709\ S^2_{bII}$
Within segments	11	708.56	64.41	S^2_{wII}
Total	25	2817.40		

$$S^2_{bII} = \frac{150.63 - 64.41}{1.709} = 50.45$$

$$C_{bII} = \$10.27 \quad C_{wII} = \$4.24$$

$$\text{opt. } \bar{n}_{II} = \sqrt{\frac{64.41}{50.45} \times \frac{10.27}{4.24}} = 1.76$$

Appendix I
(cont'd)

Winter Wheat - AOV and opt. $\bar{n}_h$ for Objective Yield

Samples in Kansas Crop Reporting Districts IV and V

District IV
AOV

Source	D.F.	S.S.	M.S.	E.M.S.
Between segments	14	2503.32	178.81	$S^2_{wIV} + 1.435\ S^2_{bIV}$
Within segments	7	632.22	90.32	S_{wIV}
Total	21	3135.54		

$$S_{bIV} = \frac{178.81 - 90.32}{1.435} = 61.67$$

$C_{bIV} = \$11.16 \quad C_{wIV} = \$3.61$

$$\text{opt. } \bar{n}_{IV} = \sqrt{\frac{90.32}{61.67} \times \frac{11.16}{3.61}} = 2.12$$

District V
AOV

Source	D.F.	S.S.	M.S.	E.M.S.
Between segments	23	2825.35	122.84	$S^2_{wV} + 1.448\ S^2_{bV}$
Within segments	11	658.55	59.87	S^2_{wV}
Total	34	3483.90		

$$S^2_{bV} = \frac{122.84 - 59.87}{1.448} = 43.49$$

$C_{bV} = \$5.93 \quad C_{wV} = \$3.31$

$$\text{opt. } \bar{n}_V = \sqrt{\frac{59.87}{43.49} \times \frac{5.93}{3.31}} = 1.57$$

Appendix I
(cont'd)

Winter Wheat - AOV and opt. $\bar{n}_h$ for Objective Yield

Samples in Kansas Crop Reporting Districts VII and VIII

District VII
AOV

Source	D.F.	S.S.	M.S.	E.M.S.
Between segments	22	4949.73	224.99	S^2_{wVII} + 1.786 S^2_{bVII}
Within segments	19	926.64	48.77	S^2_{wVII}
Total	41	5876.37		

$$S^2_{bVII} = \frac{224.99 - 48.77}{1.786} = 98.67$$

$C_{bVII} = \$6.26 \quad C_{wVII} = \$4.20$

$$\text{opt. } \bar{n}_{VII} = \sqrt{\frac{48.77}{98.76} \times \frac{6.26}{4.20}} = 0.86$$

District VIII
AOV

Source	D.F.	S.S.	M.S.	E.M.S.
Between segments	23	3937.70	171.20	S^2_{wVIII} + 1.767 S^2_{bVIII}
Within segments	19	755.02	39.74	S^2_{wVIII}
Total	42	4692.72		

$$S^2_{bVIII} = \frac{171.20 - 39.74}{1.767} = 74.40$$

$C_{bVIII} = \$7.10 \quad C_{wVIII} = \$4.32$

$$\text{opt. } \bar{n}_{VIII} = \sqrt{\frac{39.74}{74.40} \times \frac{7.10}{4.32}} = 0.94$$